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Filter Performance and Velocity Distribution Relation in Magnetic Filtration of Non-Newtonian Liquids

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ABSTRACT

Fluid flow regime and the velocity distribution are effective parameters for systems in which various transport phenomena take place. Besides the properties of the contaminating particles and filter system in the case of high gradient magnetic filtration, the rheological properties of the fluid are also important for both design and efficiency of the process. This paper presents a theoretical study about estimation of the velocity distribution in a magnetic filter and the dependence of filter performance on this distribution. A model is presented to estimate the velocity distribution and filter performance in magnetic filtration of Newtonian or weak non-Newtonian liquids. The model is essentially based on the balance of forces acting on particles captured and accumulated in the filter. Model predictions and experimental data given in the literature for Newtonian liquids are in a good agreement.

Key Words. Magnetic; Filter; Non-Newtonian liquids

INTRODUCTION

Removal of micron or submicron-sized magnetic particles (ferro, para, or diamagnetic) with very low concentrations from various technological flu-

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ids is of a great importance for several branches of industry. Due to the very small size and very low concentrations of particles, the classical filtration techniques cannot be used for fine cleaning of these kinds of fluids. High gradient magnetic filtration (HGMF) is a relatively novel technology which has been successfully used for this purpose. Due to various advantages over classical filters, HGMF has been favored in such activities as chemistry, energetics, and the machine industry (1–6). In addition, this technique has found application in recent years in scientific research in biology and medicine (7, 8).

Numerous papers are available in the literature about the mechanism and kinetics of magnetic filtration and the major parameters that affect the efficiency of the process. In most of these studies it is implicitly or explicitly assumed that the liquids subjected to magnetic filtration are Newtonian. However, recent studies have indicated that some of the technological or the biological liquids have non-Newtonian fluid properties (9). As far as the authors know, data for the magnetic filtration of non-Newtonian liquids are very limited, and the effects of non-Newtonian behavior have not been investigated.

In this paper the effects of the viscous behavior of a liquid on the velocity distribution and on the filter performance are investigated by a theoretical approach. Based on the existing models, some new relations are proposed for the estimation of filter performance in the magnetic filtration of Newtonian or non-Newtonian liquids. The validity of these relations is confirmed with experimental data given in the literature for Newtonian liquids.

DEVELOPMENT OF THE THEORY AND EQUATIONS FOR THE VELOCITY DISTRIBUTION AND FILTER PERFORMANCE

In the cleaning of Newtonian liquids by magnetic filtration, the following semiempirical relation is widely used to estimate filter performance (6):

$$\Psi/\Phi = 1 - \exp(-\alpha L) \qquad (1)$$

where Ψ is the filter performance, Φ is the magnetic particle fraction ($\Phi = C_0'/C_0$, magnetic particle concentration/total particle concentration), α is a sorption constant, and L is the filter length. The constant α characterizes the particle capture ability of the filter system and is generally determined by experimental methods. It is mainly dependent on the magnetic and geometric properties of the system as well as on the rheological properties of the liquid. Deriving a specific relation among the major system parameters which affect α is essential for predicting the theoretical filter performance. Equation (1) may be used as a basis to develop such a relationship. In addition, the following definitions or assumptions are made in this text to establish a relation between filter performance and the parameters of a filter system:

1. A liquid that contains magnetic particles is named a suspension. The magnetic particles in the suspension are of the same kind, spherical, and of equal size.
2. Total concentration of the particles (magnetic plus nonmagnetic particles) in the suspension is very low (in the range 10^{-3} ppm or lower).
3. The suspension has weak non-Newtonian properties and its rheological properties may be represented by the power law as in the Ostwald–de Waele model provided that $|n-1| \ll 1$; where n is the flow behavior index ($n = 1$ for Newtonian liquids).
4. A magnetic filter is a kind of packed bed, or a porous media, which consists of contacting magnetized ferromagnetic spherical elements with identical capacities for particle capture.
5. An active area is created around the contacting points of the filter elements in which the particles are captured and collected. The changes in the porosity of the filter due to particle capture and accumulation are negligible.
6. The main forces, which affect particle capture and accumulation, are the magnetic force (F_m) and the drag force (F_D). All other forces (e.g., gravitational, adhesion, electrical, etc.) and diffusion are neglected.
7. The velocity profiles around the spherical elements are symmetrical. A description of the velocity profile around a filter element may be sufficient to determine the velocity distribution in the flow area.

A typical cross section of fluid flow area (or pore space) in the magnetic filter is schematically demonstrated in Fig. 1. Indeed, the figure does not repre-

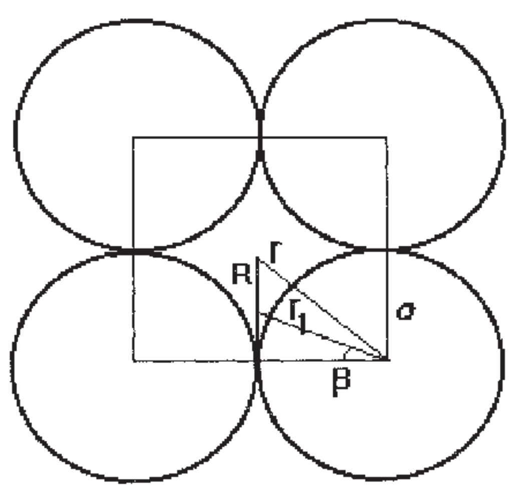

FIG. 1 Schematically representation of the fluid flow space confined by four spherical filter elements.

sent the most probable arrangement of the spheres in the bed. The natural arrangement is between cubic and rhombic, but cubic arrangement is deliberately chosen to simplify the mathematical operations. The sorption constant, α, in Eq. (1) may be estimated in terms of the system parameters which are defined in Fig. 1 as follows (6):

$$\alpha = \tfrac{3}{4}(R/a)^3/d \tag{2}$$

where R is the accumulation depth or radial coordinate from the contacting point, a is the radius of the filter element as indicated in Fig. 1, d is the diameter ($d = 2a$), and R/a is the dimensionless radius for the active area of the filter system. Equation (2) imposes that R/a must be known to estimate α. The accumulation depth, R, may be estimated by balancing the drag force and the magnetic force, i.e.,

$$F_\mathrm{m} = F_\mathrm{D} \tag{3}$$

After estimation of the effective forces, this equality may be used to derive a relation for estimation of R/a. The estimation of these forces in terms of their major parameters is summarized below.

The Magnetic Force

The magnetic force on a ferromagnetic particle in the vicinity of the contacting points of the filter elements is given by Sandulyak (6) as

$$F_\mathrm{m} = \frac{\pi\delta^3}{6}\,\frac{\mu_0(\chi_\mathrm{P} - \chi_\mathrm{f})\mu^2(\mu - 1)H^2(R/a)}{a\{1 + 0.5(\mu - 1)(R/a)^2\}^3} \cong \frac{\pi\delta^3}{2}\,\frac{\mu_0\chi\mu^{1.38}H^2}{d(R/a)} \tag{4}$$

where $\mu_0 = 4\pi \times 10^{-7}$ (H/m), the free space magnetic permeability, χ_p is the particle magnetic susceptibility, χ_f is the fluid magnetic susceptibility, χ is the effective magnetic susceptibility ($\chi = \chi_\mathrm{p} - \chi_\mathrm{f}$), μ is the relative magnetic permeability of the filter elements, H is the external magnetic field, and δ is the effective particle size.

The Drag Force

The drag force exerted on a immersed body is proportional with the velocity head and may be estimated for a spherical body by (10, 11)

$$F_\mathrm{D} = C_\mathrm{D}\pi\rho_\mathrm{f}U^2\delta^2/8 \tag{5}$$

where C_D is the drag coefficient, ρ_f is the fluid density, and U is the local flow velocity. The drag coefficient due to the flow of a Newtonian fluid is essentially a function of only the Reynolds number (Re) for a definite geometry. It may be estimated that Re is generally about Re < 1 in a magnetic filter. For the case of the flow past a sphere, C_D may be estimated from Stokes' equation or from Oseen's equation up to Re $<$ 5 (9, 10). But for a non-Newtonian liquid, it is a function of both Re and the flow behavior index, n (11, 12). Under the creeping flow conditions (Re $\ll$ 1), the drag coefficient may be estimated by (11, 12)

$$C_\mathrm{D} = 24\, X_n/\mathrm{Re}_n \tag{6a}$$

where X_n is the drag correlation coefficient, defined as

$$X_n = 3^{1.5(n-1)}\,\frac{2 + 29n - 22n^2}{n(n+2)(2n+1)} \tag{6b}$$

and $X_n(1) = 1$, which corresponds to Newtonian liquids. Re_n is the modified Reynolds number:

$$\mathrm{Re}_n = \rho_\mathrm{f}\delta^n U^{(2-n)}/K \tag{6c}$$

where K is the consistency index for a non-Newtonian liquid model.

Equations (6) may be inserted into Eq. (5) to obtain.

$$F_\mathrm{D} = 3K\pi U^n X_n \delta^{(2-n)} \tag{7}$$

The fluid velocity in Eq. (7) must be known to estimate R/a through the equality in Eq. (2). On the other side, the local velocity itself is also a function of R/a. Therefore, a relation for the dependence of the local velocity on R/a, or for the velocity distribution in the bed, must be developed to evaluate Eq. (7).

Determination of Flow Velocity Distribution in the Pores

The nature of velocity profiles in the fluids flowing in tubes or beds has been the subject of a great amount study over the years (10–12). Based on these studies, it is stated that the fluid velocity in the filter ranges from zero (at the fluid–solid interface) to a maximum value at the center of the flow space (Fig. 1). The average fluid velocity, $\langle V \rangle$, in a porous media such as a packed bed or a magnetic filter is dependent on the bulk flow velocity (V_f, the filtration velocity), the medium porosity (ε), and on n. This velocity, in general, may be estimated by following two different procedures.

In the first, the average velocity in the filter is easily calculated using the bulk flow velocity and the filter porosity (12):

$$\langle V \rangle = V_\mathrm{f}/\varepsilon^n \tag{8}$$

However, Eq. (8) is not very useful at this point since it does not show the dependence of this velocity on the velocity distribution in the bed. The local velocity must be known to estimate the drag force at a certain point in the flow area, and to solve Eqs. (4) and (7) simultaneously. Therefore, it is more useful to express the average velocity in terms of the velocity distribution.

In the second, the average velocity in the filter is calculated in terms of velocity distribution as follows (6):

$$\langle V \rangle = \frac{4}{\pi} \int_0^{\pi/4} \frac{d\beta}{(r_1 - a)} \int_a^{r_1} U\,dr \tag{9}$$

where β is a characteristic angle for a point within the particle capture zone and r_1 is the characteristic distance for that point, such that $r_1 = a/\cos\beta$, as demonstrated in Fig. 1.

The determination of the local velocity, U, or the velocity profile in a magnetic filter, by the experimental method is very difficult since the pore sizes are very small (13, 14). In fact, some analytical relations are given in the literature for the velocity distribution in non-Newtonian liquids past a single object such as a sphere or a cylinder. For example, for a liquid obeying the power law and flowing over a single sphere, the local velocities are given by (15)

$$U_r = U_\infty[1 - 1.5\,(a/r)^{1/n} + 0.5(a/r)^{3/n}]\cos\Theta \tag{10a}$$

$$U_\Theta = -U_\infty[1 - (0.75/n)(2n - 1)(a/r)^{1/n} + (0.25/n)(2n - 3)(a/r)^{3/n}]\sin\Theta \tag{10b}$$

where U_∞ is the undisturbed velocity from solid surface, and r and Θ are the polar coordinates. Theoretically, Θ varies in the range $0 \le \Theta \le 180°$ and a/r varies in the range $0 \le (a/r) \le 1$ for a single sphere. For the case demonstrated in Fig. 1, a/r varies in the $0.71 \le (a/r) \le 1$ range.

The above equations may be simplified for practical purposes by assuming $\Theta = 90°$. Then the local velocity may be estimated by the equation

$$U = (U_r^2 + U_\Theta^2)^{0.5}$$

or

$$U = |U_\Theta| = U_\infty[1 - (0.75/n)(2n - 1)(a/r)^{1/n} + (0.25/n)(2n - 3)(a/r)^{3/n}] \tag{11}$$

Equation (11) may be used to predict the velocity distribution in a non-Newtonian liquid past a single sphere. One may utilize the above relations to derive a statement to estimate the velocity profile of a non-Newtonian fluid in a packed bed made up of spherical elements.

Equation (9) is integrated to obtain $\langle V \rangle$ after substitution of Eq. (11) into this equation. Upon the integration by assuming $|n - 1| \ll 1$, the following statement is obtained after rearranging it for U_∞:

$$U_\infty = f_1(n)\langle V \rangle \tag{12a}$$

where $f_1(n)$ is a function of n:

$$f_1(n) = (25\,n)/(n + 1) \tag{12b}$$

Inserting Eq. (12) into Eq. (11) gives

$$U = f_1(n)\langle V\rangle[1 - 0.75/n)(2n - 1)(a/r)^{1/n} + (0.25/n)(2n - 3)(a/r)^{3/n}] \tag{13a}$$

This equation gives the velocity profile of a non-Newtonian liquid flowing in a packed bed. By taking into consideration that $r = (a^2 + R^2)^{0.5}$, as is seen in Fig. 1, this equation may be restated in terms of R/a:

$$U = f_1(n)\langle V\rangle[1 - (0.75/n)(2n - 1)(1 + (R/a)^2)^{-1/2n} + (0.25/n)(2n - 3)(1 + (R/a)^2)^{-3/2n}] \tag{13b}$$

The maximum velocity is observed at the center of the flow space and may be estimated by replacing $R/a = 1$ in the above equation. However, practical applications indicate that particle accumulation in the filter occurs up to $R/a = 0.3$–0.4. Based on this fact, Eq. (13) may be simplified for practical engineering applications. The statement in bracket in Eq. (13b) (or 13a) may be approximated by a simpler statement as follows provided that $R/a \leq 0.4$:

$$[1 - (0.75/n)(2n - 1)(1 + (R/a)^2)^{-1/2n} + (0.25/n)(2n - 3)(1 + (R/a)^2)^{-3/2n}] \cong 0.7/n(R/a)^{2/n} \tag{14}$$

Substitution of Eqs. (8) and (14) into Eq. (13) leads to

$$U = 0.7[f_1(n)V_f/n\varepsilon^n](R/a)^{2/n} \tag{15}$$

For magnetic filters made up of spherical elements, the porosity ε varies in the range $0.40 < \varepsilon < 0.48$. Assuming $\varepsilon = 0.44$ on the average, and inserting this value and Eq. (12) into Eq. (15) gives

$$U = f_2(n)V_f(R/a)^{2/n} \tag{16a}$$

where $f_2(n)$ is

$$f_2(n) = 17.5/[0.44^n(n + 1)] \tag{16b}$$

Equations (12) and (16) are useful statements for estimating U. For example, for a Newtonian fluid, Eq. (12) gives $U_\infty = 12.5\langle V\rangle$, Eq. (13b) gives $U_m = 4.8\langle V\rangle$ for the maximum velocity at the center of the flow space ($R/a = 1$), and Eq. (16) gives $U = 20\ V_f(R/a)^2$ for the local velocity. Based on the measurements of the velocity distribution by the laser Doppler method, the following relationship has been suggested for the local velocity in a Newtonian liquid flowing within a magnetic filter made up of spherical elements (6):

$$U = K_v V_f(R/a)^2 \tag{17}$$

where K_v is a constant with a value in the $10 < K_v < 20$ range. These results indicate that the model predictions are almost the same with experimental data given in literature for a Newtonian fluid flowing through a packed bed (6).

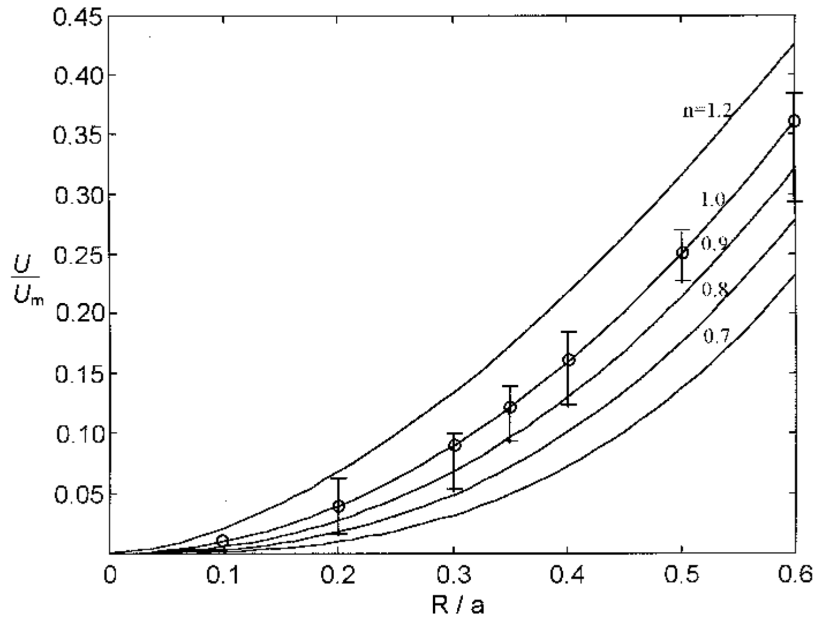

FIG. 2 The dependence of the dimensionless local velocity on the dimensionless distance from the contact points of the filter elements, and a comparison of experimental data with the model predictions. (○) Experimental data (6).

Based on Eq. (16), the dependence of the dimensionless velocity distribution (U/U_m) on R/a is demonstrated in Fig. 2 for liquids obeying the power law with n values of $0.7 \leq n \leq 1.2$. Some experimental data from the literature for a Newtonian liquid are also indicated in the figure for comparison. The figure indicates that model predictions for a Newtonian liquid are in good agreement with the experimental data. The effect of n on the velocity profiles given in the figure is in good agreement with those given in the literature (15, 16). Since filter performance is also a function of velocity (17, 18), it may be concluded that the above equations can also be used to develop some new relations for estimation of filter performance in the filtration of Newtonian or non-Newtonian liquids.

Estimation of the Filter Performance

For the prediction of filter performance, the sorption constant α must first be determined. A combination of the above Eqs. (2), (3), (4), and (7) may provide a relation among R/a, U, and filter performance. For this purpose, inserting Eq. (16) into Eq. (7) and then using Eqs. (3) and (4) gives the following relations after some mathematical arrangements:

$$(R/a)^3 = f(n)(V_m/V_f) \quad (18a)$$

where

$$f(n) = [18X_n f_2^n(n)]^{-1} \tag{18b}$$

V_m is the magnetic velocity, defined as

$$V_m = \frac{3\delta^2\mu_0\chi\mu^{1.30}H^2}{\eta_{av}d} \tag{18c}$$

η_{av} is the apparent viscosity:

$$\eta_{av} = K[V_f/\delta]^{(n-1)} \tag{18d}$$

Substitution of Eq. (18) into Eq. (2) and then into Eq. (1) gives

$$\Psi/\Phi = 1 - \exp[-0.75f(n)(V_m/V_f)(L/d)] \tag{19}$$

where L/d is the dimensionless filter length.

Equation (19) is very convenient for estimating filter performance in the magnetic cleaning of Newtonian or weak non-Newtonian liquids. For a definite filter system, L/d is a constant and therefore the performance is mainly a function of n and V_m/V_f. In the case of a Newtonian liquid, Eq. (19) simplifies to

$$\Psi/\Phi = 1 - \exp[-2.1 \times 10^{-3}(V_m/V_f)(L/d)] \tag{20}$$

Equation (20) is principally the same as Eq. (1), and indicates the effects of the main parameters of the system on performance.

Equations (19) and (20) are obtained on the assumption that $Re_n \ll 1$. In order to derive a similar relation valid for a larger range of Reynolds numbers, the lift and drag components of the drag force, instead of Eq. (5), must be taken into consideration. These components may be estimated by (17)

$$F_{Dx} = \frac{\rho_f\pi\delta^2}{4}U^{2n}\lambda_x M \tag{21a}$$

$$F_{Dy} = \frac{\rho_f\pi\delta^2}{4}U^{2n}\lambda_y M \tag{21b}$$

where λ_x and λ_y are the drag and lift components of the drag force, and M is a correlation coefficient involving the fluctuations in velocity.

Based on the effective forces, the moment balance for a particle at rest on the upper layer of the accumulated particles in the filter matrix may be given as (17)

$$F_{Dx}L_1 + F_{Dy}L_2 - F_mL_4 = 0 \tag{22}$$

where L_1, L_2, and L_4 are the distances of the application points of the forces to the touching points of the deposited particles (17). Substituting Eqs. (4), (16), and (21) into Eq. (22) with the data from Gontar (17) ($\lambda_x = 0.42$, $\lambda_y =$

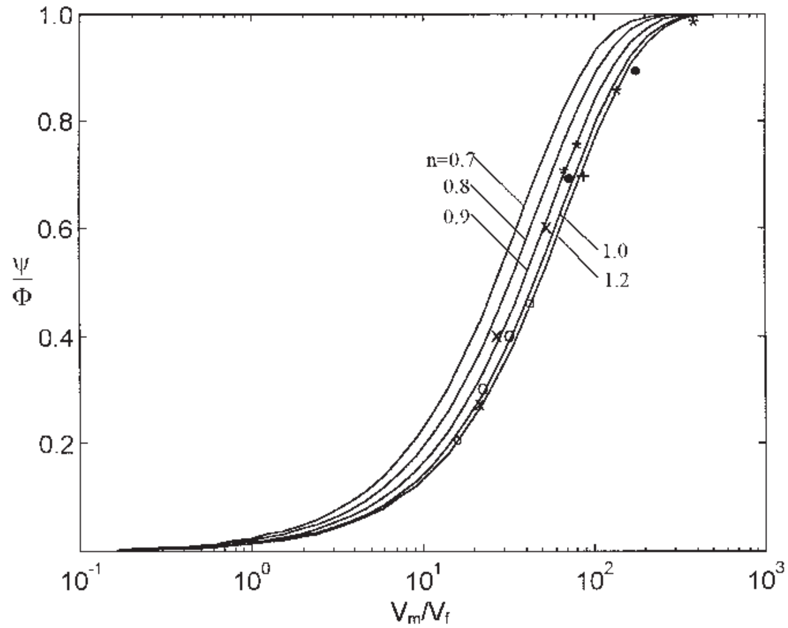

FIG. 3 The dependence of filtration performance on V_m/V_f, and a comparison of experimental data with the model predictions. Experimental data are for filtration of a suspension prepared from magnetite (Fe_3O_4) particles, $\chi = 0.4$, $d = 5.7 \times 10^{-3}$ m, $\rho_f = 1000$ kg/m^3, $L = 0.042$ m, $\Phi = 1$ (6). (○) $H = 30$ kA/m, $\delta = (3\text{–}5) \times 10^{-6}$ m; (×) $H = 125$ kA/m, $\delta = (3\text{–}5) \times 10^{-6}$ m; (□) $H = 30$ kA/m, $\delta = (7\text{–}9) \times 10^{-6}$ m; (+) $H = 30$ kA/m, $\delta = (10\text{–}15) \times 10^{-6}$ m; (●) $H = 75$ kA/m, $\delta = (10\text{–}15) \times 10^{-6}$ m; (∗) $H = 52$ kA/m, $\delta = (10\text{–}15) \times 10^{-6}$ m.

0.1, $L_1 = 0.74\delta/2$, $L_2 = 0.5\delta/2$, $L_3 = 0.5\delta/2$, $L_4 = 0.785\delta/2$, $M = 4$) enables R/a to be stated in terms of the system parameters. Substitution of the resultant statement into Eq. (2) and then into Eq. (1) gives

$$\frac{\Psi}{\Phi} = 1 - \exp\left\{-0.45[114 f_2(n)^{-2n}]^{0.6}\left(\frac{\chi \mathbf{H}^{0.75}\delta}{\rho_f V_f^2 d}\right)^{0.6}\frac{L}{d}\right\} \quad (23)$$

For the case of Newtonian liquids, this equation simplifies to

$$\frac{\Psi}{\Phi} = 1 - \exp\left[-0.212L\left(\frac{\chi \mathbf{H}^{0.75}\delta}{\rho_f V_f^2 d^{2.7}}\right)^{0.6}\right] \quad (24)$$

Equation (24) is also principally the same as Eq. (1) but indicates in detail the dependence of performance on the parameters of the system.

RESULTS AND DISCUSSION

The literature data for Newtonian liquids indicate that the sorption coefficient, α, is strongly affected by the filtration velocity. The equations devel-

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oped above suggest that the same is also true for non-Newtonian liquids. In addition, the non-Newtonian properties of a liquid also seem to have an effect on performance.

The dependence of filter performance on V_m/V_f is demonstrated in Fig. 3 for the $0.7 \leq n \leq 1.2$ range by assuming $\Phi = 1$ and $L/d = 7.36$. It is seen from the figure that the performance is appreciably dependent on V_m/V_f, especially in the $10 \leq V_m/V_f \leq 100$ range. The effect of n on performance is also more significant in this range. Some experimental data from the literature for a Newtonian liquid are also presented in the figure for the sake of comparison. These data are for the magnetic filtration of an artificially prepared suspension of Fe_3O_4 ($\Phi = 1$) in a laboratory-scale filter ($L = 0.042$ m) (6). It is seen from the figure that the model predictions and the experimental data are in good agreement. Based on this result it may be suggested that the model is valid also for non-Newtonian liquids; however, we do not have any experimental data to support this conclusion. It is seen from the figure that filter performance decreases as n increases. Theoretical calculations suggest that performance in certain filter systems may decrease up to approximately 25% as n changes from 0.7 to 1.2. This result is also in accord with literature data that the rate of mass transfer is higher in pseudoplastic fluids than in dilatants (15).

CONCLUSION

The relations developed above may be used to estimate the velocity distribution in a magnetic filter and the performance in the magnetic filtration of Newtonian or non-Newtonian liquids. The literature data for Newtonian liquids indicate that the performance of a magnetic filter is a function of various parameters (such as filtration velocity, filter length, magnetic field strength, etc.) of the system. The model developed here indicates that the same parameters are also effective in the filtration of non-Newtonian liquids. In addition, Eqs. (19) and (20), or the results in Figs. 2 and 3, suggest that viscous effects also have a substantial influence on filter performance. The performance may differ up to 25% under identical filtration conditions when the flow behavior index n varies from 0.7 to 1.2. The convenience of the model for the estimation of filter performance has been confirmed by comparing the result with experimental data given in the literature.

SYMBOLS

a	radius of the filter elements (m)
C_0	total concentration of particles before filtration (g/kg)
C_0'	total concentration of magnetic particles before filtration (g/kg)

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ΔC	decrease in the concentration after filtration
d	diameter of the filter elements (m), $d = 2a$
F_{D}	drag force (N)
F_{Dx}	drag component of the drag force (N)
F_{Dy}	lift component of the drag force (N)
F_{m}	magnetic force (N)
HGMF	high gradient magnetic filtration
H	external magnetic field (A/m)
$\mathbf{H}$	dimensionless magnetic field strength
K	flow consistency index ($\mathrm{Pa \cdot s}^n$)
K_{v}	a constant for estimating the velocity profile within a porous filter
L	filter length in flow direction (m)
L_1, L_2, L_4	characteristic distances as defined in Ref. 17 (m)
M	a correlation coefficient regarding the fluctuations in velocity
n	flow behavior index for fluids obeying the power law
r	distance of center of fluid flow space to the center of the filter element (m) (Fig. 1)
r_1	distance of a point within the fluid flow space to the center of the filter element (m) (Fig. 1)
R	radial coordinate from the contacting point of the filter elements (m)
Re_n	modified Reynolds number
U	local interstitial fluid velocity at distance R (m/s)
U_{m}	maximum interstitial velocity (m/s)
U_∞	undisturbed fluid velocity (m/s)
V_{m}	magnetic velocity as defined in Eq. (20c) (m/s)
V_{f}	filtration velocity (bulk flow velocity) (m/s)
$\langle V \rangle$	average interstitial fluid velocity (m/s)
X_n	drag correlation coefficient

Greek Letters

α	a sorption constant (m^{-1})
β	a characteristic angle for a point within fluid flow space (Fig. 1)
δ	effective particle size (m)
ε	filter porosity
λ_{x}	a coefficient for drag components of the drag force
λ_{y}	a coefficient for lift components of the drag force
ρ_{f}	fluid density ($\mathrm{kg/m^3}$)
μ	magnetic permeability of the filter elements
μ_0	the free space magnetic permeability $= 4\pi \times 10^{-7}$ (H/m),

η_{av}	apparent viscosity as defined in Eq. (20d) (Pa·s)
χ_p	particle magnetic susceptibility
χ_f	fluid magnetic susceptibility
χ	effective magnetic susceptibility ($= \chi_p - \chi_f$)
Ψ	filter performance ($= \Delta C/C_0$)
Φ	fraction of magnetic particles in the fluid ($= C_0'/C_0$)

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